The dynamics of high and low pressure systems
Newton’s second law for a parcel of air in an inertial coordinate system

(a coordinate system in which the coordinate axes do not change direction and are not accelerated)

\[
\frac{D\vec{V}}{Dt} = -\frac{1}{\rho} \nabla P + \vec{g} + \vec{F}
\]

- acceleration of a parcel of air of unit mass
- gravity
- pressure gradient force
- friction force
What is the effect of using this non-inertial coordinate system on the form of Newton’s second law?

Answer
We must add correction terms to account for variations in the direction of the coordinates.

In scalar form the equations of motion for each direction become:

(East-West equation)
\[
\frac{Du}{Dt} - \frac{uv \tan \phi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + F_x
\]

(North-South equation)
\[
\frac{Dv}{Dt} + \frac{u^2 \tan \phi}{a} + \frac{vw}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + F_y
\]

(Up-Down equation)
\[
\frac{Dw}{Dt} - \frac{u^2 + v^2}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + F_z - g
\]
What are the correction terms and what do they mean?

Consider third equation:

$$\frac{Dw}{Dt} = \frac{u^2 + v^2}{a} + \text{forces}$$

Air moving in a straight line at a constant speed initially southward.

\[ u = 0, \; v = -10 \; \text{m/s}, \; w = 0 \]

Air will accelerate upward!

The earth curves away from the path of the air parcel.
Motion on a rotating earth – the Coriolis Force

In the absence of a twisting force called a torque, air in motion across the earth must conserve its angular momentum \((M \times V \times R)\) where \(M\) is the parcel mass, \(V\) is velocity about the axis of rotation and \(R\) is the distance from the axis of rotation.

A simple interpretation of the Coriolis effect:

Air moving across the earth’s surface will try to come to equilibrium at a latitude/altitude where its angular momentum equals that of the earth so that the parcel has no relative motion.
Motion on a rotating earth – the Coriolis Force

Consider air moving:

**Eastward**: The air has greater angular momentum than the earth beneath it. It will experience an “outward” centrifugal acceleration equatorward.

**Westward**: The air has less angular momentum than the earth beneath it. It will experience an “inward” centripetal acceleration poleward.

**Poleward**: The air will progressively move over points on the earth with less angular momentum. Air will accelerate eastward relative to the earth below it.

**Equatorward**: The air will progressively move over points on the earth with greater angular momentum. Air will accelerate westward relative to the earth below it.
Motion on a rotating earth – the Coriolis Force

In a reference frame on the earth the Coriolis effect appears as a force acting on an air parcel.

The Coriolis Force

Causes air to deviate to the right of its direction of motion in the Northern Hemisphere (and to the left in the Southern Hemisphere);

Affects the direction an object will move across the earth’s surface, but has no effect on its speed;

Is strongest for fast-moving objects and zero for Stationary objects; and

Has no horizontal component at the equator and has a maximum horizontal component at the poles
What is the effect of the earth’s rotation on the form of Newton’s second law?

**Answer**

We must add terms to account for the acceleration required for air to conserve its angular momentum.

In scalar form the equations of motion for each direction become:

(East-West equation)

\[
\frac{Du}{Dt} + \frac{uv \tan \phi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + F_x + 2\Omega v \sin \phi - 2\Omega w \cos \phi
\]

(North-South equation)

\[
\frac{Dv}{Dt} + \frac{u^2 \tan \phi}{a} + \frac{vw}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + F_y - 2\Omega u \sin \phi
\]

(Up-Down equation)

\[
\frac{Dw}{Dt} + \frac{u^2 + v^2}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + F_z - g + 2\Omega u \cos \phi
\]
The complete momentum equations on a curved rotating earth

\[ \frac{Du}{Dt} - \frac{uv \tan \phi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + F_x + 2\Omega v \sin \phi - 2\Omega w \cos \phi \]

\[ \frac{Dv}{Dt} + \frac{u^2 \tan \phi}{a} + \frac{vw}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + F_y - 2\Omega u \sin \phi \]

\[ \frac{Dw}{Dt} - \frac{u^2 + v^2}{a} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + F_z - g + 2\Omega u \cos \phi \]

FOR SYNOPTIC SCALE MOTIONS…..

WHICH TERMS ARE LARGE AND IMPORTANT?
WHICH TERMS ARE SMALL AND INSIGNIFICANT?
ABOVE THE BOUNDARY LAYER

\[ \frac{Du}{Dt} - \frac{uv \tan \phi}{a} + \frac{w}{a} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nabla \cdot \mathbf{F}_x + 2\Omega v \sin \phi - 2\Omega u \cos \phi \]

\[ \frac{Dv}{Dt} + \frac{u^2 \tan \phi}{a} + \frac{vw}{a} = - \frac{1}{\rho} \frac{\partial P}{\partial y} + \nabla \cdot \mathbf{F}_y - 2\Omega u \sin \phi \]

\[ \frac{Dw}{Dt} - \frac{u^2 + v^2}{a} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + g - g + 2\Omega u \cos \phi \]

WITHIN THE BOUNDARY LAYER

\[ \frac{Du}{Dt} - \frac{uv \tan \phi}{a} + \frac{w}{a} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + F_x + 2\Omega v \sin \phi - 2\Omega u \cos \phi \]

\[ \frac{Dv}{Dt} + \frac{u^2 \tan \phi}{a} + \frac{vw}{a} = - \frac{1}{\rho} \frac{\partial P}{\partial y} + F_y - 2\Omega u \sin \phi \]

\[ \frac{Dw}{Dt} - \frac{u^2 + v^2}{a} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + g - g + 2\Omega u \cos \phi \]
The Hidden Simplicity of Atmospheric Dynamics:

\[
\frac{Du}{Dt} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + fv
\]

\[
\frac{Dv}{Dt} = - \frac{1}{\rho} \frac{\partial P}{\partial y} - fu
\]

\[
\frac{Du}{Dt} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + F_x + fv
\]

\[
\frac{Dv}{Dt} = - \frac{1}{\rho} \frac{\partial P}{\partial y} + F_y - fu
\]

\[
0 = - \frac{1}{\rho} \frac{\partial P}{\partial z} - g
\]

ABOVE THE BOUNDARY LAYER, ALL HORIZONTAL PARCEL ACCELERATIONS CAN BE UNDERSTOOD BY COMPARING THE MAGNITUDE AND DIRECTION OF THE PRESSURE GRADIENT AND CORIOLIS FORCES

WITHIN THE BOUNDARY LAYER, ALL HORIZONTAL PARCEL ACCELERATIONS CAN BE UNDERSTOOD BY COMPARING THE MAGNITUDE AND DIRECTION OF THE PRESSURE GRADIENT, CORIOLIS AND FRICTIONAL FORCES

THE ATMOSPHERE IS IN HYDROSTATIC BALANCE – VERTICAL PGF BALANCES GRAVITY – ON SYNOPTIC SCALES
A state of balance between the pressure gradient force and the Coriolis force

Air is in geostrophic balance if and only if air is not accelerating (speeding up, slowing down, or changing direction).

For geostrophic balance to exist, isobars (or height lines on a constant pressure chart) must be straight, and their spacing cannot vary.

\[ 0 = \frac{Du}{Dt} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + fv \]

\[ 0 = \frac{Dv}{Dt} = -\frac{1}{\rho} \frac{\partial P}{\partial y} - fu \]
Geostrophic wind

\[ u_g = -\frac{1}{\rho f} \frac{\partial P}{\partial y} \quad \text{and} \quad v_g = \frac{1}{\rho f} \frac{\partial P}{\partial x} \]

The wind that would exist if air was in geostrophic balance

The Geostrophic wind is a function of the pressure gradient and latitude
Pressure coordinates

The same equations can be derived in pressure coordinates instead of height coordinates.

In pressure coordinates, the horizontal momentum equations (above the boundary layer) become:

\[
\frac{Du}{Dt} = - \frac{\partial \Phi}{\partial x} + f\nu \\
\frac{Dv}{Dt} = - \frac{\partial \Phi}{\partial y} - fu
\]

where \( \Phi \), the geopotential height, is given by \( \Phi = gz \)

The geostrophic wind becomes:

\[
u_g = - \frac{1}{f} \frac{\partial \Phi}{\partial y} \\
v_g = \frac{1}{f} \frac{\partial \Phi}{\partial x}
\]

And the hydrostatic balance equation becomes

\[
\frac{\partial \Phi}{\partial P} = - \frac{R_d T}{P}
\]
Implications of Geostrophic Balance

\[ u_g = -\frac{1}{f} \frac{\partial \Phi}{\partial y} \quad \text{and} \quad v_g = \frac{1}{f} \frac{\partial \Phi}{\partial x} \]

Take P derivative:

\[ \frac{\partial u_g}{\partial P} = -\frac{1}{f} \frac{\partial}{\partial y} \left( \frac{\partial \Phi}{\partial P} \right) \quad \text{and} \quad \frac{\partial v_g}{\partial P} = \frac{1}{f} \frac{\partial}{\partial x} \left( \frac{\partial \Phi}{\partial P} \right) \]

Substitute hydrostatic eqn

\[ \frac{\partial \Phi}{\partial P} = -\frac{R_d}{fP} \frac{\partial T}{\partial y} \quad \text{and} \quad \frac{\partial \Phi}{\partial P} = -\frac{R_d}{fP} \frac{\partial T}{\partial x} \]

THE RATE OF CHANGE OF THE GEOSTROPHIC WIND WITH HEIGHT (PRESSURE) WITHIN A LAYER IS PROPORTIONAL TO THE HORIZONTAL TEMPERATURE GRADIENT WITHIN THE LAYER.
Consider the following scenario where there is a horizontal temperature contrast:

We will define the **thermal wind** as the vertical shear of the geostrophic wind.

Combining the equations on the previous slide back into vector form:

\[
\frac{\partial u_g}{\partial P} = \frac{R_d}{f p} \frac{\partial T}{\partial y} \quad \frac{\partial v_g}{\partial P} = -\frac{R_d}{f p} \frac{\partial T}{\partial x} \quad \Rightarrow \quad \frac{\partial V_g}{\partial p} = -\frac{R_d}{f p} \mathbf{k} \times \nabla T
\]

We define \(-\frac{\partial V_g}{\partial p}\) as the **thermal wind**.  

\[
\frac{\partial V_g}{\partial p} = V_{gU} - V_{gL}
\]
Starting with \( \frac{\partial V_g}{\partial p} = -\frac{R_d}{fp} \mathbf{k} \times \nabla T \), now use hydrostatic eqn.:

\[
\frac{\partial p}{\partial z} = -\rho g \quad \rightarrow \quad \frac{g \partial z}{\partial p} = -\frac{1}{\rho} = -\frac{RT}{p}, \quad g \partial z = \partial \phi \quad \rightarrow \quad \frac{\partial \phi}{\partial p} = -\frac{RT}{p}
\]

\[
V_T = -\frac{\partial V_g}{\partial p} = \frac{R_d}{fp} \mathbf{k} \times \nabla T \quad \text{now becomes} \quad V_T = -\frac{\partial V_g}{\partial p} = \frac{1}{f} \mathbf{k} \times \nabla \frac{\partial \phi}{\partial p}
\]

The thermal wind in a layer blows parallel to the thickness contours, with low thicknesses to the left.
The thermal wind relations also tells us about temperature advection by the geostrophic wind. In the example at left, the winds are turning counterclockwise with height. The average wind in the layer ($\mathbf{V}_{\text{AVG}}$) is performing CAA.

Thus, counterclockwise turning of the winds with height (backing) is associated with geostrophic $\text{CAA}$.

Clockwise turning of the wind with height (veering) is associated with geostrophic $\text{WAA}$. 

1000-500 hPa thicknesses
Note the position of the front at 850 mb......
and the jetstream at 300 mb.
More steeply sloped pressure surfaces imply that a stronger pressure gradient will exist aloft, and therefore a stronger geostrophic wind.
…..but where on this 300 mb map is air actually in geostrophic balance?
Extratropical cyclones develop and evolve in atmospheric conditions that are not in geostrophic balance!
Geostrophic imbalances above the boundary layer and their impact on sea level pressure
The effect of curvature of a low pressure pattern on the balance of forces:

Assume air at “A” is in geostrophic balance.

With no net force acting on the parcel it will move differentially forward at a constant speed. For clarity, we will allow it to move to “B”

At “B”, a component of the PGF acts opposite the motion, reducing the parcel speed and thus reducing the Coriolis force.

The PGF exceeds the Coriolis force creating a centripetal acceleration down the pressure gradient toward the low pressure center.

Air curves to the left, following the isobars and going into orbit around the low pressure center.
Curved flow around low pressure center

1) Air velocity is sub-geostrophic \( V < V_g \)

2) The pressure gradient force exceeds the Coriolis force at all points in the flow pattern.

3) Air accelerates toward the low forcing it’s velocity to change in a manner that leads it to orbit around the low.

4) Winds flow parallel to the isobars

5) For pure circular flow, the centripetal acceleration is given by \( V^2/R \), where \( V \) is the rotational velocity component and \( R \) is the distance to the center of rotation.
The effect of curvature of a high pressure pattern on the balance of forces:

Assume air at “A” is in geostrophic balance.

With no net force acting on the parcel it will move differentially forward at a constant speed. For clarity, we will allow it to move to “B”.

At “B”, a component of the PGF acts in the direction of the motion, increasing the parcel speed and thus increasing the Coriolis force.

The Coriolis exceeds the PGF force creating a centripetal acceleration down the pressure gradient toward the high pressure center.

Air curves to the right, following the isobars and going into orbit around the high pressure center.
1) Air velocity is super-geostrophic \( V > V_g \)

2) The Coriolis force exceeds the pressure gradient force at all points in the flow pattern.

3) Air accelerates toward the high forcing its velocity to change in a manner that leads it to orbit around the high.

4) Winds flow parallel to the isobars

5) For pure circular flow, the centripetal acceleration is given by \( \frac{V^2}{R} \), where \( V \) is the rotational velocity component and \( R \) is the distance to the center of rotation.
Curved flow: Divergence and convergence in a ridge/trough pattern

Note: In the pattern above, the isobars are equally spaced implying that the pressure gradient force is uniform along the flow channel.

\[ V_a = \frac{1}{f} \hat{k} \times \left( \frac{DV}{Dt} \right) \]
Jetstreaks

Suppose that air is in geostrophic balance at Point A

As air moves toward point B, the PGF increases. Air will accelerate toward low pressure since the PGF exceeds the Coriolis force.

Suppose that air is in geostrophic balance at Point C

As air moves toward point D, the PGF decreases. Air will accelerate toward high pressure since the Coriolis force exceeds the PGF.
Effect of acceleration of air through a jetstreak on convergence and divergence patterns

\[ V_a = \frac{1}{f} \hat{k} \times \frac{DV}{Dt} \]

Green arrows denote flow of air if the air was in geostrophic balance.

Blue arrows denote deflection of air because air is not in geostrophic balance.
Vertical circulations about entrance and exit region of a jetstreak

Thermally direct circulation (turns Potential Energy into Kinetic Energy)

Thermally indirect circulation (turns Kinetic Energy into Potential Energy)

(Important for upper level frontogenesis)
The combined effects of curvature changes and jetstreaks on convergence and divergence in a trough ridge system where a jetstream is located at the base of the trough.

The maximum divergence aloft is located on the northeast side of the trough within the left exit region of the jetstreak. The maximum convergence aloft is located on the northwest side of the trough within the left entrance region of the jetstreak.
Consider the phasing together of the divergence and convergence patterns associated with ageostrophic accelerations due to flow curvature and jetstreaks as a jetstreak propagates through the base of a trough in the jetstream.

At what point in this progression will a low pressure center rapidly deepen at the surface?

C = convergence
D = divergence
Blue = curvature
Red = jetstreak
Coupled jetstreak circulations can enhance the development of a low pressure center
The Boundary Layer

Layer of air adjacent to the earth’s surface that is significantly influenced by the force of friction

\[
\frac{Du}{Dt} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + F_x + fv
\]

\[
\frac{Dv}{Dt} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + F_y - fu
\]

Friction always acts opposite the direction of flow reducing the velocity of the flow
Friction:
- acts through viscous forces in a narrow layer near the earth’s surface (collision between air and ground transfers momentum to earth which is dissipated as heat)
- in the rest of the atmosphere, acts by mixing of high momentum air with low momentum air due to mechanical, thermal and shear-induced turbulence.
Example of a well mixed boundary layer

Dry adiabatic lapse rate

Constant mixing ratio

00Z 16 Mar 1994

University of Wyoming
Effect of friction on force balances in lower atmosphere

Net result:
Friction causes air to flow across isobars from high to low pressure in the Boundary layer

The amount of turning and decrease in wind speed depends on surface roughness
Effect of friction on surface high and low pressure systems

DYNAMIC PROCESSES IN THE BOUNDARY LAYER ALWAYS ACT TO DESTROY LOW AND HIGH PRESSURE SYSTEMS
Note cross isobaric flow
Force balances associated with strengthening highs or deepening lows
The “isallobaric wind” effect

For simplicity, assume flow is initially in geostrophic balance.

Short time ($\Delta t$) later, low deepens a small amount ($\Delta P$) (exaggerated for clarity), increasing PGF.

Air will accelerate down pressure gradient until balance is reestablished.

Net effect: In regions of developing highs or deepening lows, air will have a component of flow from high to low pressure due to accelerations associated with the pressure changes.
Effect of diabatic heating (left) and diabatic cooling (right) on the development of highs and lows.

A. Suppose we have a layer of air with uniform temperature in the horizontal.

B. Let's heat the layer in the center through the release of latent heat during condensation so that temperature of layer increases and pressure surfaces rise.

C. An outward directed pressure gradient force will develop aloft. Air will evacuate the column, restoring balance aloft, but creating low pressure at the surface.

D. Let's cool the layer in the center by radiating energy to space at night so that temperature of layer decreases and pressure surfaces fall.

E. An inward directed pressure gradient force will develop aloft. Air will fill the column, restoring balance aloft, but creating high pressure at the surface.
This process is: essential in hurricanes, Asian and Amer. Monsoon, significant in oceanic extratropical cyclones, less important in continental extratropical cyclones.

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**Diagram B:**
- Let's heat the layer in the center through the release of latent heat during condensation so that temperature of layer increases and pressure surfaces rise.
- Layer heating.
- Pressure levels: 500 mb, 600 mb, 700 mb, 800 mb, 900 mb, 1000 mb.

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**Diagram C:**
- An outward directed pressure gradient force will develop aloft.
- Air will evacuate the column, restoring balance aloft, but creating low pressure at the surface.
- Pressure levels: 500 mb, 600 mb, 700 mb, 800 mb, 900 mb, 1000 mb.
This process is: essential for all major high pressure systems
Dynamic processes (the jetstreak and curvature effects associated with force imbalances) and thermodynamic processes (heating and cooling) lead to the redistribution of mass in the atmosphere and create the world’s low and high-pressure systems and its jetstreams. In nature, dynamic and thermodynamic processes work simultaneously.
When divergence occurs within a column of air, the surface pressure will decrease with time, and a low-pressure center will develop.

*Divergence is associated with changes in the curvature in the flow, jetstreaks, and heating of the atmospheric column.*

Once a low-pressure system forms at the surface, air will circulate around the low, but the force of friction and the isallobaric effect will cause the air to turn toward the low, converging into the low-pressure center.

Friction-induced convergence will work to destroy the low, since it “fills” the atmospheric column.

The low will intensify (pressure will drop) if the divergence aloft exceeds the convergence near the surface.

The low will weaken (pressure will rise) if the opposite occurs.
Air converges into an air column aloft to form a high-pressure system.

Convergence associated with diabatic cooling is generally more important than convergence associated with curvature and jetstreak processes. Cooling occurs over broad areas, such as the North Atlantic and Pacific Oceans in summer, or the Canadian and Asian Arctic in winter - the areas where strong high-pressure systems develop.

High-pressure systems are enhanced by convergence associated with the jetstream as these cool airmasses move out of their source regions.

Once a high forms, air flows clockwise around a high, and outward away from the high due to friction. The divergence near the surface associated with friction acts to destroy the high-pressure system (by causing pressure to lower).

The high will intensify (pressure will rise) if the convergence aloft exceeds the divergence near the surface.

The high will weaken (pressure will fall) if the opposite occurs.