

Derive Eq. (5.33), an equation that governs time-dependent, inviscid changes in $\xi = \partial u / \partial z - \partial w / \partial x$, the component of vorticity in the y -direction. Identify the rhs terms, and use simple illustrations to describe the physical processes represented by these terms.

[Note that there are many different starting points for this problem. One is to take $\partial / \partial z$ of $Du/Dt = -1/\rho \partial p / \partial x + fv$ (Eq. (2.16)) and subtract from it $\partial / \partial x$ of $Dw/Dt = -1/\rho \partial p / \partial z - g$ (similarly scaled version of Eq. (2.18)), bearing in mind that $D/Dt = \partial / \partial t + \vec{V} \cdot \nabla$. Because no *a priori* assumptions have been made about density and pressure, a solenoidal term of the form

$$-\frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial z} - \frac{\partial \rho}{\partial z} \frac{\partial p}{\partial x} \right),$$

will appear on the rhs of the resulting equation. If we now impose the thermodynamic variable decompositions (i.e., Eq. (2.21)), assume that the base state is in hydrostatic balance, and recall that products of deviation state variables are negligibly small, this term will reduce to

$$-\frac{\partial B}{\partial x} + \frac{1}{\bar{\rho}^2} \frac{d\bar{\rho}}{dz} \frac{\partial p'}{\partial x},$$

where $B = -g\rho'/\bar{\rho}$. The first term is sometimes referred to as the *thermal solenoid* (e.g., Ziegler et al. 1995), and appears explicitly in Eq. (5.33). The second term is referred to as the *pressure solenoid*. It is neglected here (as it is in many similar applications), on the grounds that $d \ln \bar{\rho} / dz$ in a typical atmosphere is extremely small.]