

Numerically integrate the 1D linear advection equation (or linear wave equation)

$$\frac{\partial u}{\partial t} = -c \frac{\partial u}{\partial x} , \quad (1)$$

using the *leapfrog scheme*,

$$u_i^{n+1} = u_i^{n-1} - c \frac{\Delta t}{\Delta x} (u_{i+1}^n - u_{i-1}^n) , \quad (2)$$

and the *upstream* (forward-in-time, forward-in-space) *scheme*,

$$u_i^{n+1} = u_i^n - c \frac{\Delta t}{\Delta x} (u_i^n - u_{i-1}^n) , \quad (3)$$

Assume periodic boundary conditions  $u(0,t) = u(L,t)$ , and an initial condition

$$u(x,0) = c + A \sin(kx) , \quad (4)$$

with  $c = 2 \text{ m s}^{-1}$ ,  $A = 1 \text{ m s}^{-1}$ ,  $\Delta x = 200 \text{ m}$ ,  $k = 2\pi/L$ , and  $L = 10 \Delta x$ . Choose two different time steps, one of which satisfies the CFL condition  $|c\Delta t / \Delta x| \leq 1$ , and one that exceeds the CFL condition. Plot the resulting solution after 1, 20, 50 time steps (and more/less as needed).

Using the same procedure (and time steps) as before, numerically integrate Eq. (1) with the *Euler explicit* (forward-in-time, centered-in-space) *scheme*

$$u_i^{n+1} = u_i^n - c \frac{\Delta t}{2\Delta x} (u_{i+1}^n - u_{i-1}^n) , \quad (5)$$

which is unconditionally unstable.