

Show that during the early stage of supercell development, the net circulation

$$\Gamma = \oint \vec{V} \cdot d\vec{l} ,$$

about the midlevel updraft is zero in the general case of unidirectional environmental vertical wind shear.

Hint: Choose an integration circuit that encloses the vertical-vortex couplet and hence over an area such as $(-1 \leq x \leq 1, 0 \leq y \leq 1)$, and then use a velocity field of the form

$$\begin{aligned} u(x, y = 0) &= \sin \pi x , \\ u(x, y = 1) &= -\sin \pi x , \\ v(x = -1, y) &= \sin \pi y , \\ v(x = 1, y) &= \sin \pi y . \end{aligned}$$