As mentioned in Section 6.3, the negative thermal buoyancy of convective outflow can be diluted by surface fluxes of heat and moisture. This effect is represented in the simple axisymmetric density-current model given by Ross et al. (2004):

$$\frac{dr}{dt} = \hbar\sqrt{g'd} \qquad (1)$$

$$\frac{dg'}{dt} = \frac{C_d}{d}(u_0 + u)(g_s' - g') \qquad (2)$$

where u ($\equiv dr/dt$) is the radial velocity of density current, r is the radial distance from the current source, d is the characteristic current depth, $\hbar$ is a coefficient (set here $=\sqrt{2}$; see Chapter 2), u_0 is the mean background wind, C_d is a dimensionless aerodynamic drag, and where

$$g' = g\frac{\theta_0 - \theta}{\theta_0} \qquad (3)$$

is the reduced gravity (see Chapter 2), or negative thermal buoyancy. In Eqs. (2)-(3), subscript s indicates a surface value, and subscript 0 indicates a background or environmental value. Eqs. (1)-(2) can be combined into a single differential equation and then integrated upon assuming that the density current speed is slow compared to the background wind (i.e., $u << u_0$). The solution (given as Eq. 10 in Ross et al. 2004) can be used to determine the radial distance at which $g' = 0$ owing to surface heat and moisture fluxes:

$$r_{crit} \approx \left[\frac{8\sqrt{2}\mathcal{V}^{3/2}g_i'^{1/2}}{C_d u_0 \pi^{3/2}}\left(1 - \alpha\tan^{-1}\frac{1}{\alpha}\right)\right]^{1/4} \qquad (4)$$

where $\mathcal{V}$ is the (assumed constant) volume of the density current, g_i' is the initial reduced gravity, and $\alpha = (-g_s'/g_i')^{1/2}$; g_s' is assumed to be constant and < 0 for a warm surface. This critical radius, or "run-off distance," is the focus of this exercise.

Following Ross et al. (2004), let us consider values representative of tropical oceanic conditions and convection: θ_0 = 300 K, θ_s = 298 K, θ_i = 297 K (the initial temperature of the convective outflow) and u_0 = 5 m s^{-1}. Find r_{crit} if $C_d = 1.3 \times 10^{-3}$ and $\mathcal{V} = 1.1 \times 10^{11}$ m^3. Double the value of $\mathcal{V}$ to get a volume closer to that of midlatitude convection, let θ_i = 295 K, and then re-compute r_{crit}. Experiment with a range of drag coefficient values and surface temperatures. Comment on the process(es) negelected in this simple model, and how these might change the results.