Homework #1 – due 12:00 PM Friday, Feb. 8, 2018

- You are encouraged to work with others on this assignment,
  ... but *what you hand in must be your own work. No copying allowed!*
- Also: • solutions *must* be handwritten; no typing! • no late submissions accepted
- Provide your answers on separate sheets of paper - not this one.

1. Consider this equation: $\dfrac{\partial^2 u}{\partial x^2} + xy\dfrac{\partial^2 u}{\partial x \partial y} - 2x^2 u\dfrac{\partial^2 u}{\partial y^2} = 0$

   a) Is this differential equation linear? Why or why not?

   b) What is the order of this differential equation? _______

2. Regarding the Lotka-Volterra solutions we looked in class:

   a) What kinds of errors were apparent? (brief description!)

   b) Based on the plots that were shown in class, under what circumstances would the method we used converge to the correct solution?

3. Consider the 2-D staggered (Arakawa C) grid, with scalar variables at a grid box *center*. The velocity variables are located at **(choose one):**

   a. same location as scalar (e.g. pressure, temperature) variables

   b. parallel-flow sides of the grid box (U is staggered +/- $\Delta y/2$)

   c. normal-flow sides of the grid box (V is staggered +/- $\Delta y/2$)

4. Consider this FTCS numerical scheme: $\dfrac{\left(q_j^{n+1} - q_j^n\right)}{\Delta t} + c\,\dfrac{\left(q_{j+1}^n - q_{j-1}^n\right)}{2\Delta x} = 0$

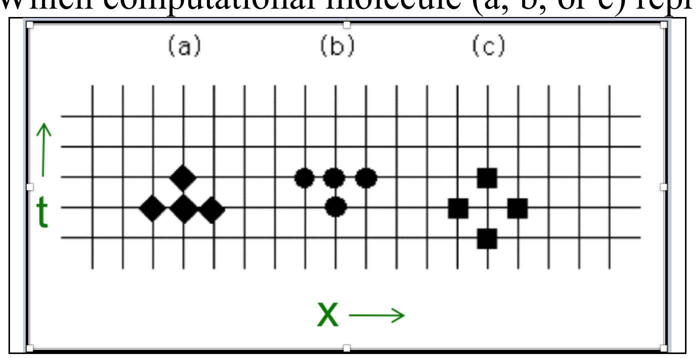
   a) Derive the truncation error.

   b) State the order of accuracy – i.e. order $\{(\Delta x)^m, (\Delta t)^n\}$: what are m, n?

   c) Is it *consistent* with the PDE, $\dfrac{\partial q}{\partial t} + c\dfrac{\partial q}{\partial x} = 0$? How do you know?

   d) Use von Neumann's method to show this is absolutely unstable
   (assume the Courant number *and* nondimensional wavenumber are nonzero)

   e) Which computational molecule (a, b, or c) represents this method?



5. The answer to the question "*how do wavenumber-dependent phase errors distort the solution?*" requires <u>two</u> parts.
    a. We first suppose that any solution can be represented as an infinite series of … *(finish this sentence)*
    b. We then say that, if individual wavenumber components traveled at different phase speeds, then … *(finish this sentence)*

6. Study the following polar plots carefully. For ν=0.75, estimate:
    a. the *approximate* amplification factor for 2Δx <u>and</u> 4Δx waves.
    b. the *approximate* phase error (ratio, $\frac{\phi}{\phi_e}$) for 2Δx <u>and</u> 4Δx waves.