Exam 1 review

Following is a list of what could be on the exam. The relevant material includes class material, handouts, Homework #1, and class discussions through/including Thu. Feb. 7. *Note: All that you need to know about papers – journal handouts – are that which was <u>covered in the notes and in class</u>!*

1. Be able to explain in words and/or in diagrams/plots why the nonlinear 1-d wave equation problem (Program #1 case C) produces sharp gradients at the center when initialized with a simple sine wave. No equations allowed!

2. Linear vs. nonlinear PDEs –
   a. how we distinguish between them; and, what is *nonlinear transition* ?
   b. know the 1-D "advection equation" (also called the 1-way wave equation) in linear <u>*and*</u> nonlinear forms.

3. Solutions to the one-way wave equation
   a. How did our solution fail in program #1 case B? In particular: How did the horizontal scales (in wavelength <u>*and*</u> wavenumber) of the solution *at the end* compare with the scale *in the initial condition* for that problem?

4. What do we mean by a "3-point stencil" when discussing a numerical method?

5. Be able to figure out – and write down code for on the exam – periodic boundary conditions in a 1-D problem.

6. Be able to identify forward, backward and centered time differencing.

7. Be aware of general issues raised in Predator-Prey (Lotka-Volterra) example:
   a. What approximations were made (i.e. to what *types* of terms?)
   b. What tradeoff was present when we sought a more accurate answer?
   c. Don't memorize the equations (in physical or discrete) form. I'm most interested in what we learned about a physical problem.

8. Be able to write down the (first few terms of the) Taylor series expansion of some expression, e.g. $f(t+\Delta t)$, $f(x-\Delta x)$, etc.

9. What *is* truncation / discretization error?
   a. Be able to derive it.
   b. How do we derive the *modified* equation? What is the purpose of finding the modified equation for a scheme: what does the modified equation tell us about the leading error terms that we don't see immediately from the truncation error?
   c. How do we determine the *order of accuracy* from the truncation error or the modified equation?
   d. How do we determine the *dominant type of errors* given the truncation error or modified equation results?
   e. What is *consistency?*

10. Stability
    a. What do we mean by a numerical scheme being *unstable*?
    b. Be able to derive the amplification factor and stability condition
        i. I will tell you exactly what kind of form I want for the answer, e.g. whether "0 < sigma < b" is sufficient or if I want it written in terms of (for example) the time step.
    c. Given the amplification factor λ,
        i. Be able to say which wavelength or *k*Δ*x* will go unstable first ($|\lambda|$ largest) by examination of the amplification factor λ.
        ii. Describe the amplitude behavior vs. time (beyond "stable" vs. "unstable": oscillating? Damping?)
        iii. Describe the amplitude and phase properties of infinitely long waves, or nΔx waves (e.g. 2Δx or 4Δx).

11. Stagggered grids
    a. What *is* a staggered grid (in words)?
    b. Be able to explain *"A-grid"* and *"C-grid"*.

12. What do we mean when we say, for the von Neumann stability condition, that every resolvable Fourier component must be *bounded*?

13. Norms:
    a. Know the L1, L2 and infinite vector norms

14. Dispersion:
    a. Be able to explain (2-part answer!) how *wavenumber-dependent phase speeds* can cause short-wavelength oscillations as we saw behind a traveling cone or square wave in 1-D advection
        i. these oscillations *are errors* for the linear 1-way wave equation: everything should move at the speed **c** from the 1-way wave equation.

15. Be able to interpret diagrams such as:
    a. Takacs-like, polar plots or other plots describing the amplitude or phase errors for a scheme, e.g. as a function of the nondimensional wavenumber $k\Delta x$ and the Courant number ν

16. When we do a von Neumann stability analysis of the 1-D linear advection or diffusion equations, we end up with some kind of stability condition depending on coefficients. For example,

$$\left(\frac{|c|\Delta t}{\Delta x}\right) \leq some\ number;\ \left(\boldsymbol{K}\frac{\Delta t}{(\Delta x)^2}\right) \leq some\ number$$

• *What does this tell* you about how the time step Δt depends on:
    a. magnitude of the flow speed |*c*|, grid spacing Δ*x*, diffusion coefficient **K**?

17. The 1-D *flow configuration* and boundary conditions used in Program 1 and the 2-D setup in Program 2 had advantages to them. We picked that flow / BC for what reason??

18. Takacs paper
    a. His error equations: what does linear correlation coefficient ρ tell us regarding dissipation vs. dispersion error?
    b. How increasing accuracy (say, 1st to 2nd or 2nd to 4th order) affects the errors in phase or amplitude
    c. Which type of order-accuracy in space yields better (lower) amplitude error - odd or even order? Which is better for phase error?
    d. Takacs' scheme: started with a 3-point-stencil, and then he added one additional grid point.
        i. How did he determine the coefficient of this point?
        ii. Where was this point located? (j-2 or j+2: how did he choose?)

19. What are some limitations on use of the von Neumann stability analysis? (e.g., it applies only if … – or it proves to be a necessary and sufficient condition for stability only if … what??)

20. Know general properties of even- and odd-order accuracy [spatial] schemes, i.e. relationship between order-*accuracy* of scheme, *dominant type* of error to be expected (dissipation/dispersion), <u>and</u> the odd/even-order accuracy of the *spatial derivative* in the leading modified equation error term.

21. Know our operator definitions such as $\delta_x f$ and $\overline{f}^{2x}$
    a. be able to do simple expansion of this notation for averaging, derivatives.
    b. be able to suggest differencing methods for a problem, as we did in class with a simple 1-D staggered grid layout.

22. *Implicit diffusion* (also in notes as *implicit viscosity*).
    a. What is it? [not *implicit* as in derivatives at (n+1)]
    b. What is the main "problem" with relying on it to provide background diffusion, when modeling?

23. Know about:
    a. Dispersion, dissipation, and diffusion -- given a step-function initial state, know how dissipation vs. dispersion would change the solution.
    b. Shift condition

24. Know about:
    a. Wavelength and wavenumber
    b. Discretization or truncation error
    c. Stability and consistency
    d. Courant number, in terms of the 1-way wave equation
    e. Grid *size (dimensions)* vs. grid *spacing* vs. *resolution*
    f. Forward, backward, and centered time differencing
    g. That the amplification factor describes error *in <u>one</u> time step*
    h. Dispersion, dissipation, and diffusion
    i. L1, L2, and infinite norms *for vectors*

25. Implicit vs. Explicit methods
    a. advantages, disadvantages of each
    b. how to recognize them
    c. how implicit advection methods stay stable with Courant number > 1 ?

Not on the exam:

- Any of the bazillion equations in Takacs' paper.
- Exact form of Takacs' numerical method (or of *any scheme*!)
  or of Predator-Prey equations
- Deriving schemes from Taylor series (we'll do that soon!)
- Aspects of paper/journal articles other than what is mentioned in this review
- The material in the notes that we *did not* get covered in class on Thursday Feb. 7 – that is: split vs. unsplit operators, Smagorinsky's "Crowley extension" material, and the section at the end re: directional splitting.